Review

Last time we looked at fundamental trig identities, including: reciprocal, quotient and Pythagorean.

Fundamental Trigonometric Identities

<table>
<thead>
<tr>
<th>Reciprocal Identities</th>
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<tbody>
<tr>
<td>$\sin \theta =$</td>
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<td>$\csc \theta =$</td>
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<tr>
<th>Quotient Identities</th>
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<td>$\tan \theta =$</td>
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<tr>
<th>Pythagorean Identities</th>
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<tr>
<td>$\sin^2 \theta + \cos^2 \theta = 1$</td>
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Example: Use the fundamental trig identities to verify the following trig identity:

$$(\sec \theta + \tan \theta)(1 - \sin \theta) = \cos \theta$$
Section 4.4 (and 4.2)  Trigonometric Functions of Any Angle (trig with the unit circle)

We want to extend the definition of the trig function to any angle (not just acute). To this end, let $\theta$ be any angle in standard position with $(x, y)$ a point on the terminal side of $\theta$. Then by the Pythagorean theorem, the distance from $(x, y)$ to the origin is $r = \sqrt{x^2 + y^2}$. Then we have the following definitions for the trig functions of $\theta$:

Example 1: Let $(-2,3)$ be a point on the terminal side of $\theta$. Find the sin cosine and tangent of $\theta$:

Unit Circle  Recall that a circle of radius one centered at the origin is given by the equation $x^2 + y^2 = 1$. Now if $(x, y)$ is any point on the unit circle, then $1 = r = \sqrt{x^2 + y^2}$, so
Then we can use certain points on the unit circle to help us remember the values of the trig functions at special angles such as $\frac{\pi}{3}$, etc. The Unit Circle $x^2 + y^2 = 1$

**Example 2:** Evaluate the trig functions at each real number:

(a) $\theta = \frac{\pi}{6}$  
(b) $\theta = \frac{5\pi}{4}$  
(c) $\theta = \frac{5\pi}{3}$  
(d) $\theta = -\frac{\pi}{6}$
Properties of the Trigonometric Functions

The domain of the sine and cosine is all real numbers (why?). Also, for any point \((x, y)\) on the unit circle, \(-1 \leq x \leq 1\) and \(-1 \leq y \leq 1\). Thus, it follows that \(-1 \leq \sin \theta \leq 1\) and \(-1 \leq \cos \theta \leq 1\).

Now adding \(2\pi\) to \(\theta\) gives another rotation around the unit circle to the same point, so \(\sin \theta = \sin (\theta + 2\pi)\). Also, \(\cos \theta = \cos (\theta + 2\pi)\). Thus, we say that sine and cosine are periodic with period \(2\pi\).

Recall that a function \(f\) is even if \(f(-x) = f(x)\) and \(f\) is odd if \(f(-x) = -f(x)\).

Even Trig Functions:

Odd Trig Functions:

Example 3: Use the above properties to help evaluate

\[
\begin{align*}
(a) \ \sin \left(\frac{13\pi}{6}\right) & \quad (b) \ \cos \left(-\frac{\pi}{4}\right) & \quad (c) \ \sin (-t) \text{ if } \sin t = \frac{4}{5}
\end{align*}
\]
Example 4: Give that \( \cot \theta = -\frac{3}{4} \) and \( \sin \theta > 0 \), find \( \cos \theta \) and \( \csc \theta \).

Reference Angles

Let \( \theta \) be an angle in standard position. Then its reference angle is the acute angle \( \theta' \) formed by the terminal side of \( \theta \) and the horizontal axis.

Example 5: For each of the following, find the reference angle:

(a) \( \frac{7\pi}{6} \)  
(b) \( 200^\circ \)  
(c) \( 400^\circ \)

Procedure for Evaluating Trig Functions at any Angle \( \theta \)

1. Find the value of the trig function at the corresponding reference angle \( \theta' \)-the value of the trig function at \( \theta \) is the same as the value at \( \theta' \), except for possibly the sign.
2. You can determine the correct sign for your trig function by finding the quadrant in which \( \theta \) lies and the sign of your trig function in that quadrant.

Example 6: Evaluate the trig functions using reference angles:

(a) \( \sin \left( \frac{5\pi}{3} \right) \)  
(b) \( \sec (-210^\circ) \)  
(c) \( \tan \left( \frac{1.5\pi}{4} \right) \)
Example 7: Let $\theta$ be an angle in Quadrant 2 with $\tan \theta = \frac{3}{2}$. Find $\sec \theta$.

Example 8: Find two solutions of $\cos \theta = -\frac{\sqrt{2}}{2}$ for $0 \leq \theta < 2\pi$. 